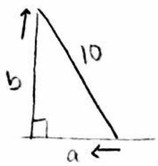
Lesson 16: Related Rates, Part 2

1. The base of a 10-ft ladder that is leaning against a wall is pushed towards the wall. When the base is 6 ft from the wall and moving at the rate of 2 ft/sec, how fast is the top of the ladder sliding up the wall?

① 

$a^2+b^2=10^2$ not changing!

② $a=6,\ \frac{da}{dt}=-2$ $\quad 6^2+b^2=10^2$

Want: $\frac{db}{dt}$ $\quad b=\sqrt{10^2-6^2}=8$

③ $\cancel{2}a\frac{da}{dt}+\cancel{2}b\frac{db}{dt}=0$

④ $6(-2)+8\,\frac{db}{dt}=0$

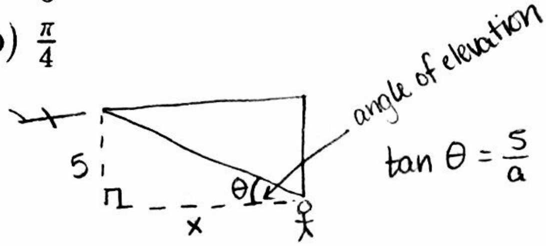
⑤ $\frac{db}{dt}=\frac{12}{8}=\frac{3}{2}$

⑥ $\boxed{\frac{3}{2}\text{ ft/sec}}$

2. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation is changing when the angle is:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{4}$

(a) ① 

$\tan\theta = \frac{5}{a}$ OR $\cot\theta = \frac{x}{5}$

② $\frac{dx}{dt} = -600$, $\theta = \frac{\pi}{3}$ OR

Want: $\frac{d\theta}{dt}$

③ $-\csc^2\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

④ $-\left(\frac{2}{\sqrt{3}}\right)^2 \frac{d\theta}{dt} = \frac{1}{5}(-600)$ OR

⑤ $\frac{d\theta}{dt} = \frac{600 \cdot 3}{20} = 90$

⑥ $\boxed{90 \text{ rad/hr}}$

(b) $\theta = \frac{\pi}{4}$

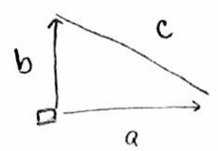
$-\left(\frac{\sqrt{2}}{1}\right)^2 \frac{d\theta}{dt} = \frac{1}{5}(-600)$

$\frac{d\theta}{dt} = \frac{600}{10} = 60$

$\boxed{60 \text{ rad/hr}}$

3. Two airplanes depart the Purdue Airport. One leaves at noon heading due east at 550 miles per hour and the other leaves at 12:30pm heading due north at 600 miles per hour. How quickly is the distance between them changing at 1:30pm?

1



$a^2+b^2=c^2$

2 $\frac{da}{dt} = 550, \quad \frac{db}{dt} = 600,$

at 1:30:

$a = 1.5(550), \quad b = 1(600), \quad c = \sqrt{(1.5(550))^2 + 600^2}$

$= 825$

$= \sqrt{1040625}$

$= 25\sqrt{1665}.$

Want: $\frac{dc}{dt}$

3 $\cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt} = \cancel{2}c\frac{dc}{dt}$

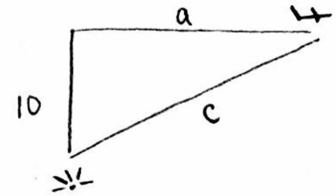
4 $825(550) + 600(600) = 25\sqrt{1665}\,\frac{dc}{dt}$

5 $\frac{dc}{dt} = \frac{825(550) + 600(600)}{25\sqrt{1665}}$

6 $= \boxed{\frac{32550}{\sqrt{1665}} \text{ miles/hr}}$

4. An airplane flying at an altitude of 10 miles passes directly over a radar antenna. When the airplane is 15 miles away, the radar detects that the distance is changing at a rate of 250 miles per hour. What is the speed of the airplane?

(1)



$a^2 + 10^2 = c^2$

(2) $c = 15, \quad \frac{dc}{dt} = 250 \longrightarrow$ $\sqrt{15^2 - 10^2} = \sqrt{125} = 5\sqrt{5}$

Want: $\frac{da}{dt}$

(3) $\cancel{2}a \frac{da}{dt} = \cancel{2}c \frac{dc}{dt}$

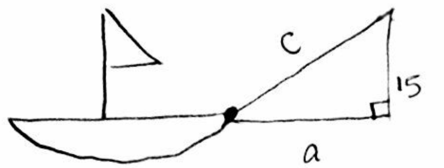
(4) $5\sqrt{5} \frac{da}{dt} = 15(250)$

(5) $\frac{da}{dt} = \frac{750}{\sqrt{5}} = 150\sqrt{5}$

(6) $\boxed{150\sqrt{5} \text{ miles/hour}}$

5. A boat is pulled into a dock by means of a winch 15 feet above the deck of the boat. The winch pulls in rope at a rate of 5 feet per second. Determine the speed of the boat when there is 39 feet of rope out.

1



$a^2 + 15^2 = c^2$

2 $\frac{dc}{dt} = -5$, $c = 39$ $\longrightarrow$ $a = \sqrt{39^2 - 15^2} = 36$

Want: $\frac{da}{dt}$

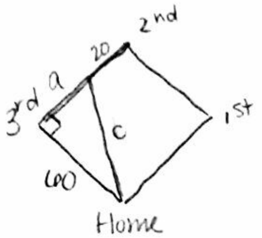
3 $\cancel{2}a \frac{da}{dt} = \cancel{2}c \frac{dc}{dt}$

4 $36 \frac{da}{dt} = 39(-5)$

5 $\frac{da}{dt} = \frac{-65}{12}$

6 $\boxed{\frac{65}{12} \text{ ft/sec}}$ (since speed is positive)

6. In softball, the distance between each base is 60 feet. A player is running from second base to third base at a speed of 16 feet per second. Find the rate at which the distance from home plate is changing when the player is 20 feet from second base.

① 

$a^2 + 60^2 = c^2$

② $\frac{da}{dt} = -16, \quad a = 60 - 20 = 40 \rightarrow$ 40, 60, $\sqrt{5200} = 20\sqrt{13} = c$

Want: $\frac{dc}{dt}$

③ $\cancel{2}a \frac{da}{dt} = \cancel{2}c \frac{dc}{dt}$

④ $40(-16) = 20\sqrt{13} \frac{dc}{dt}$

⑤ $\frac{dc}{dt} = \frac{-36}{\sqrt{13}}$

⑥ $\boxed{-\frac{36}{\sqrt{13}} \text{ ft/sec}}$